Adaptive Beamforming of ESPAR Antenna Using Sequential Perturbation

Jun Cheng, Yukihiro Kamiya, and Takashi Ohira

ATR Adaptive Communications Research Laboratories, Kyoto 619-0288 Japan

Abstract **A technique for adaptively controlling the loaded reactances on the passive radiators, thus forming both beam and nulls is presented, for the first time, for the electronically steerable passive array radiator (ESPAR) antenna. Conventional adaptive array antenna processing must access signals on all of the array antenna elements. However, because the low-cost ESPAR antenna only has a single-port output, all of the signals on the antenna elements cannot be observed. The adaptive algorithm proposed in this paper is based on the steepest gradient theory, where the reactances are sequentially perturbed to determine the gradient vector. Simulations show that the ESPAR antenna can be adaptive. The statistical performance of the output SIR of the ESPAR antenna is also given.**

1. Introduction

The adaptive array antenna is an emerging technology that has gained much attention over the last few years for its ability to significantly increase the performance of wireless systems by effectively suppressing cochannel interference [1]. While the wireless community has paid a great deal of attention to adaptive systems for basestations, relatively little effort has been focused on how to improve *mobile terminals* using advanced adaptive antenna techniques. Mobile terminal applications also have stringent limitations on hardware and algorithm complexity as well as power consumption. It is well known that nearly all-existing antenna array techniques also require one receiver chain per branch of antenna. For a two-element array antenna, for instance, this doubles the receiver hardware and is not an easy trade-off to make where reducing system complexity is essential. For the reasons of hardware, complexity, power consumption, etc., no array antenna, which multiplies the receiver hardware by number of element branches, is a suitable candidate for low-power mobile terminal applications.

Analog adaptive beamformers, e.g., an electronically steerable passive array radiator (ESPAR) antenna [2] [3], have shown the potential for application to wireless communication systems, and especially to *mobile terminals*. The $(M+1)$-element ESPAR antenna, as depicted in Fig. 1 where $M = 6$, has *only an active radiator* (the 0-th element) connected to the receiver. The remaining M elements are passive, and the antenna pattern is formed according to the values of the loaded *reactances* on these passive radiators.

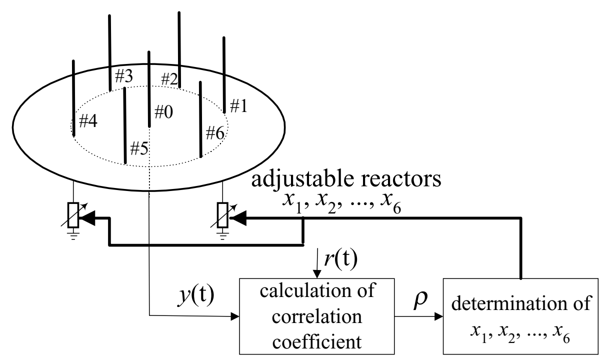

Figure 1: Diagram of an adaptive ESPAR antenna.

Because of the configuration of the ESPAR antenna, we may face the following three difficulties with the development of the adaptive algorithm.

(a) For the ESPAR antenna, the signals on the surrounding passive elements ($\#1 \sim \#M$) cannot be observed. Only the single-port output can be observed and processed as feedback to adjust the reactances.

(b) The RF current on each element is not independent but mutually coupled with each other, and depends on the value of the reactances.

(c) The single-port output is a highly nonlinear function of these *variable reactances*, and includes an intractable matrix inverse (see Eq. (3) below).

Recalling the conventional adaptive array, the received signal on each element is observed. The weight coefficient in each element branch is independently controlled. The antenna output is a linear function of the complex weight coefficients. Hence, a direct application of most of the algorithms for the conventional adaptive array to the ESPAR antenna is impractical. It is desirable to propose adaptive algorithms especially for the ESPAR antenna.

Although several adaptive algorithms for single-port antennas have been proposed, they are still not sufficient for application to mobile communication systems. In [2] [4] and [5], an adaptive algorithm of control reactances based on the steepest descent method was proposed. However, it was designed mainly so as to cancel interferences so that the beam steering was not sufficiently examined.

In this paper, our objective is to develop an adaptive algorithm that makes the ESPAR antenna steer its beam and nulls automatically, thus making it self-adaptive. The proposed algorithm is based on the steepest gradient theory, where the values of the reactances are sequentially perturbed to determine the gradient vector. By this adaptive algorithm, the loaded reactances are adjusted to null out or at least reduce the source of interferences, thus to make the SIR (signal-to-interference ratio) as large as possible. The development of the adaptive algorithm makes the ESPAR antenna adaptive for the first time.

2. ESPAR Antenna Formulation

An $(M+1)$-element ESPAR antenna [3] with $M = 6$ is depicted in Fig. 1. The 0-th element is an active radiator located at the center of a circular ground plane. It is a $\lambda/4$-monopole (where λ is the wavelength) and is excited from the bottom in a coaxial fashion. The remaining M elements of $\lambda/4$-monopoles are passive radiators surrounding the active radiator symmetrically with the radius $R = \lambda/4$ of the circle. Each of these M elements is terminated by a *variable reactance*. Thus adjusting the values x_m $(m = 1, 2, \cdots, M)$ of the reactances can change the pattern of the antenna. The vector denoted by

$$\boldsymbol{x} \triangleq [x_1, x_2, \cdots, x_M]^{\mathrm{T}} \tag{1}$$

is called the *reactance vector*, where the superscript $^{\mathrm{T}}$ is the transpose of the vector or matrix.

Denote $\boldsymbol{s}(t) = [s_0(t), s_1(t), \cdots, s_M(t)]^{\mathrm{T}}$, where the component $s_m(t)$ is the RF signal impinging on the m-th element of the ESPAR antenna. Then, the single-port RF output $y(t)$ of the antenna is given by

$$y(t) = \boldsymbol{i}^{\mathrm{T}}\boldsymbol{s}(t), \tag{2}$$

where $\boldsymbol{i} = [i_0, i_1, \cdots, i_M]^{\mathrm{T}}$ is the RF current vector with the component i_m appearing on the m-th element [6].

According to the electromagnetic analysis of the ESPAR antenna, the RF current vector $\boldsymbol{i}$ is formulated as [6]:

$$\boldsymbol{i} = (\boldsymbol{I} + \boldsymbol{Y}\boldsymbol{X})^{-1}\boldsymbol{y_0}, \tag{3}$$

where $\boldsymbol{I}$ is the identity matrix of order $M + 1$, and the diagonal matrix $\boldsymbol{X} = \mathrm{diag}[50, jx_1, jx_2, \cdots, jx_M]$ is called the reactance matrix. Moreover, in Eq. (3), the $(M+1)$-dimensional vector $\boldsymbol{y}_0$ and $(M+1)$ by $(M+1)$ matrix $\boldsymbol{Y}$ are constant determined by the mutual admittance between the elements.

It should be emphasized that the signal vector $\boldsymbol{s}(t)$ in Eq. (2) impinging on the elements of the ESPAR antenna is not measurable. This differs from the conventional adaptive array where the received signal vector on the elements is observed. For the ESPAR antenna, only the single-port output $y(t)$ can be measured, and used as feedback to control the reactance vector $\boldsymbol{x}$ of Eq. (1). More unfortunately, as shown in Eq. (3), the single-port output $y(t)$ is a highly nonlinear function of $\boldsymbol{x}$, and includes an intractable matrix inverse, which makes it difficult to produce an analytical expression of adaptive performance. It is also interesting to note that the current vector $\boldsymbol{i}$ in Eq. (3) is equivalent to the weight coefficient vector of conventional adaptive array. It is clear from Eq. (3) that each component of $\boldsymbol{i}$, unlike the weight coefficient vector of the conventional adaptive array, is not independent but mutually coupled with each other. The discussion above implies a direct application of most of the algorithms of the conventional adaptive array to the ESPAR antenna is impractical. It is desirable to propose an adaptive algorithm especially for the ESPAR antenna.

3. Signal Model

Suppose there are a total number of $Q + 1$ signal sources transmitting signals $u_q(t)$ with DOAs θ_q $(q = 0, 1, \cdots, Q)$. Let $s_m(t)$ $(m = 0, 1, \cdots, M)$ denote the signal impinging on the m-th element of the antenna, and let $\boldsymbol{s}(t)$ be the column vector with m-th component $s_m(t)$. Then, the column vector $\boldsymbol{s}(t)$ may be expressed as

$$\boldsymbol{s}(t) = \sum_{q=0}^{Q} \boldsymbol{a}(\theta_q)u_q(t). \tag{4}$$

Here $\boldsymbol{a}(\theta_q)$ is the steering vector defined by

$$\boldsymbol{a}(\theta_q) = \begin{bmatrix} 1 \\ e^{j\frac{\pi}{2}\cos(\theta_q - \varphi_1)} \\ e^{j\frac{\pi}{2}\cos(\theta_q - \varphi_2)} \\ \vdots \\ e^{j\frac{\pi}{2}\cos(\theta_q - \varphi_M)} \end{bmatrix}, \tag{5}$$

where $\varphi_m = \frac{2\pi}{M}(m-1)$ $(m=1,\cdots,M)$. Substituting Eq. (4) into Eq. (2) yields the output of the ESPAR antenna

$$y(t) = \boldsymbol{i}^{\mathrm{T}}\boldsymbol{s}(t) = \sum_{q=0}^{Q} \boldsymbol{i}^{\mathrm{T}}\boldsymbol{a}(\theta_q)u_q(t). \tag{6}$$

Notice that the current vector $\boldsymbol{i}$, and thus $y(t)$ is a function of the reactance vector $\boldsymbol{x}$ of Eq. (1).

4. Adaptive Algorithm

We are now ready to describe a gradient-based adaptive algorithm of the ESPAR antenna. In this algorithm, a reference signal $r(t)$ is used, which is assumed to be known to both the transmitter and the receiver. Abusing notation slightly, in the remaining part of this paper we still denote by $y(t)$ the equivalent lowpass signal of the RF output of the ESPAR antenna.

In the conventional steepest gradient algorithm, the commonly used cost function is the mean-squared error. For ESPAR antenna, however, the amplitude of the each component in the current vector $\boldsymbol{i}$ of Eq. (3) seems not to be sensitive to the reactance vector $\boldsymbol{x}$. This means the amplitude of $y(t)$ can not be freely controlled by the reactance vector. For this reason, an amplitude adjustment of $y(t)$ is needed if the mean-squared error is used as the cost function.

Instead of the mean-squared error, in our algorithm a *cross-correlation coefficient* is adopted. It is well known that the cross-correlation coefficient represents the similarity of two signals, while the error represents the difference. Assume $\boldsymbol{y}(n)$ and $\boldsymbol{r}(n)$ are the P-dimensional column vectors that are discrete time samples of $y(t)$ and $r(t)$, respectively. The cross-correlation coefficient of $\boldsymbol{y}(n)$ and $\boldsymbol{r}(n)$ at time n is defined as

$$\rho_n = \frac{|\boldsymbol{y}^{\mathrm{H}}(n)\boldsymbol{r}(n)|}{\sqrt{\boldsymbol{y}^{\mathrm{H}}(n)\boldsymbol{y}(n)}\sqrt{\boldsymbol{r}^{\mathrm{H}}(n)\boldsymbol{r}(n)}}, \quad (7)$$

where superscript $^{\mathrm{H}}$ denotes a complex conjugate transpose. Then a gradient vector is defined as

$$\nabla\rho_n \triangleq \frac{\partial\rho_n}{\partial\boldsymbol{x}} \triangleq \begin{bmatrix} \frac{\partial\rho_n}{\partial x_1} \\ \frac{\partial\rho_n}{\partial x_2} \\ \vdots \\ \frac{\partial\rho_n}{\partial x_M} \end{bmatrix}, \quad (8)$$

where $\frac{\partial\rho_n}{\partial\boldsymbol{x}}$ denotes the derivative with respect to reactance vector $\boldsymbol{x}$.

As it is well known, the output SIR is represented as $\rho_n^2/(1-\rho_n^2)$ if the desired signal is not correlated with the interference signals. This implies that the larger the cross-correlation coefficient, the larger the output SIR. Our goal is to find a reactance vector $\boldsymbol{x}$ of (1) such that the cross-correlation coefficient, thus the output SIR is as large as possible. By the steepest gradient algorithm, we proceed as follows:

(i) Begin with an initial value $\boldsymbol{x}$ of the reactance vector, which is chosen arbitrarily. Typically, $\boldsymbol{x}$ is set equal to the zero vector, where the initial antenna pattern is omni-directional.

(ii) Using this initial or present guess, compute the gradient vector $\nabla\rho_n$ at time n (i.e., the n-th iteration).

(iii) Calculate the next guess at the reactance vector by making a change in the initial or present guess in a direction similar to that of the gradient vector.

(iv) We go back to step (ii) and repeat the process.

It is intuitively reasonable that successive corrections to the reactance vector in the directions of the positive of the gradient vector should eventually lead to a good reactance vector in the sense that the cross-correlation coefficient is large.

Let $\boldsymbol{x}(n)$ denote the value of reactance vector $\boldsymbol{x}$ of Eq. (1) at time n. According to the steepest gradient method described above, the update value of the reactance vector at time $n+1$ is computed by using the simple recursive relation

$$\boldsymbol{x}(n+1) = \boldsymbol{x}(n) + \mu\nabla\rho_n \quad (9)$$

where μ is a positive real-value constant that controls the convergence speed.

There may be some difficulty when we compute the gradient vector $\nabla\rho_n$ of Eq. (8). This arises from, as we have stated in Section 2, the fact that a) it may not be easy to analytically represent the gradient vector as a function of $\boldsymbol{x}$ because of the presence of the intractable matrix inverse in the representation of $y(t)$ (see Eqs. (2) and (3)); and b) the signal vector impinging on the elements of the ESPAR antenna can not be observed.

An estimate of the gradient vector $\nabla\rho_n$ of Eq. (8) may be derived by the use of finite-difference approximations of derivatives [7] [8]. Specifically, the first-order partial derivative $\partial\rho_n/\partial x_i$ with respect to the reactance x_i is approximated to a change of the cross-correlation coefficient

$$\begin{aligned} \frac{\partial\rho_n}{\partial x_m} \approx [&\rho_n(x_1, x_2, \cdots, x_m + \Delta x_m, \cdots x_M) \\ &-\rho_n(x_1, x_2, \cdots, x_m, \qquad \cdots x_M)]/\Delta x_m, \quad (10) \\ &m = 1, 2, \cdots, M, \end{aligned}$$

by incrementing x_m to $x_m + \Delta x_m$. Substituting this estimate of the gradient vector into Eq. (9), the reactance vector $\boldsymbol{x}(n+1)$ is calculated. Repeating these steps from $n = 1$ to $n = N$, we obtain, for a sufficiently large N, a good reactance vector $\boldsymbol{x}(N+1)$ in the sense that the cross-correlation coefficient ρ_N is large.

As shown in Eq. (10), only one component of the gradient vector $\nabla\rho_n$ is calculated at a time from the output of the antenna. All the components of reactance vector $\boldsymbol{x}$ are sequentially perturbed in order to get one gradient vector for each iteration of Eq. (9). This sequential perturbation of the reactance requires $M+1$ times transmission of the reference vector $\boldsymbol{r}(n)$ (with length P) for one iteration. Thus, the total $P(M+1)N$ symbols is required for N iterations.

5. Simulations

The presence of an intractable matrix in the output representation (see Eqs. (2) and (3)) of the ESPAR antenna may make it difficult to describe its performance analytically. Simulations are required to validate the proposed algorithm and the antenna behaviors. In our simulations, a 7-element ($M = 6$) ESPAR antenna is used. We chose the powers of all the source signals $u_q(t)$ ($q = 0, 1, \cdots, Q$) to be unity. The absence of noise is assumed for the reason of simplicity. In all the experiments, the data block size for each calculation of the cross-correlation coefficient defined in Eq. (7) is taken to be $P = 10$.

Let first consider the case where there are two signals from different directions. After $N = 800$ iterations, the beam is steered to 0° of the desired signal, while the deeper null is formed towards the interference signal at 135° (see Fig. 2(a)). The output SIR of 28.26dB is obtained. Fig. 2(b) shows the convergence curve associated with the forming of the pattern of Fig. 2(a). The number of symbols used for training are $P(M+1)N = 10 \times (6+1) \times 800 = 56000$.

Second, we consider the statistical performance of the output SIR of the adaptive ESPAR antenna. Fig. 3 illustrate the probability $Pr(Z \geq z)$ that the output SIR of Z will exceed a given real number z of the abscissa. In the calculation for this figure, the desired signal is fixed to come from an angle of 0°, and the DOA(s) of the interference signals are set to be uniformly random in the range of 0° to 359°. All 1000 sets of DOAs are used in these statistics. The curves are given in the cases of number $Q = 1$, 2, 3 and 4 of interference signal(s). As an example of how to interpret these curves, Fig. 3 (in the case of $Q = 4$) implies that the adaptive antenna can provide at least 20dB output SIR (in other words 26.02dB SIR gain) with a probability of 80%.

6. Conclusions

In this paper, a steepest gradient-based algorithm is proposed for the ESPAR antenna, where the reactances are sequentially perturbed to determine the gradient vector. The simulation results show that a 7-element ESPAR antenna can provide an SIR gain of at least about 26dB with a probability of 80%. The development of the adaptive algorithm makes the low-complexity ESPAR antenna adaptive for the first time. This shows that the ESPAR antenna is an excellent candidate for practical antenna implementations in wireless mobile terminals.

In this paper the absence of noise is assumed for the reason of simplicity. The performance simulations in noise deserve further investigation for a practical purpose. Development of an adaptive algorithm with fast convergence remains as another topic for further research on ESPAR antenna.

References

[1] A.J. Paulraj and C.B. Papadias, "Space-time processing for wireless communications," IEEE Signal Processing Magazine, Nov. 1997, pp. 49-83.

[2] R.J. Dinger and W.D. Meyers, "A compact HF antenna array using reactively-terminated parasitic elements for pattern control," Naval Research Laboratory Memorandum Report 4797, May 1982.

[3] T. Ohira and K. Gyoda, "Electronically steerable passive array radiator antennas for low-cost analog adaptive beamforming," 2000 IEEE Inter. Conf. on Phased Array System & Technology, pp. 101-104, Dana Point, California, May 21-25, 2000.

[4] R.J. Dinger, "Reactively steered adaptive array using microstrip patch elements at 4GHz," IEEE Trans. on Antennas and Propagation, vol. AP-32, no. 8, pp. 848-856, Aug. 1984.

[5] R.J. Dinger, "A planar version of a 4.0 GHz reactively steered adaptive array," IEEE Trans. on Antennas and Propagation, vol. AP-34, no. 3, pp. 427-431, Mar. 1986.

[6] T. Ohira, K. Gyoda, A. Akiyama and K. Yang, "Equivalent weight vector and array factor formulation for Espar antennas," Technical Report of IEICE, AP2000-44, SAT2000-41, NW2000-41, July 2000. (in Japanese)

[7] R.A. Monzingo and T.W. Miller, *Introduction to adaptive arrays*, John Wiley & Sons, Inc., 1980.

[8] John Van de Vegte, *Feedback Control System*, Third Edition, Prentice Hall, 1994.

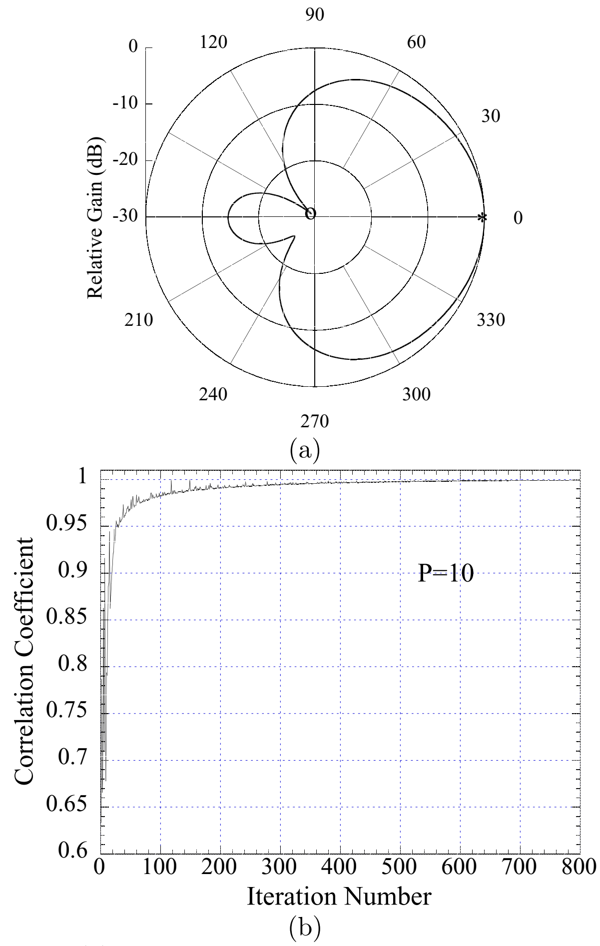

Figure 2: (a) Antenna pattern after $N = 800$ iterations. DOAs of desired signal and interference signals are: 0° and 135°, respectively. Input SIR: 0dB, Output SIR: 28.26dB. (b) Convergence curve.

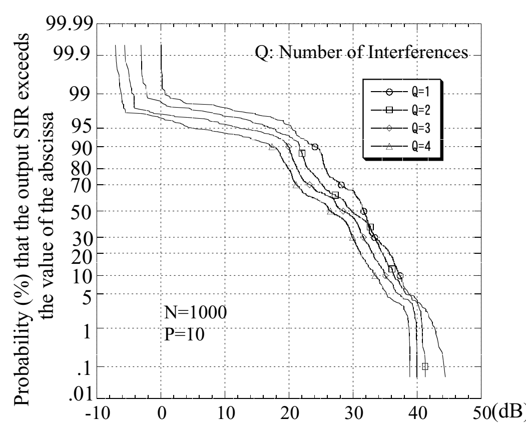

Figure 3: Probability $Pr(Z \geq z)$ that output SIR of Z exceeds a given real number z of the abscissa after iterations of $N = 1000$.